at the same time. The method works directly with the hybrid set of differential equations . . . ."

The assertion, that the method is the new one, is not quite right. The method has been developed first in papers[2,3] (see also the monograph[4]) for applying to the problem on stability of solid bodies with liquid-filled cavities. The hybrid set of the differential equations of the system motion has been also obtained with the help of Hamilton's principle. The energy or a combination of the energy and the first integrals of the hybrid set of differential equations has been considered as Liapunov function and functional simultaneously.

This method is indeed the general and rigorous one and should be applicable to the stability analysis in many areas.

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Reply by Author to V. V. Rumyantsev

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IN his Comment, Rumyantsev disputes the assertion made in Ref. 1 that "A new method of approach to the stability problem of hybrid systems, . . . , is presented." He argues the point by contending that the "general and rigorous" method was developed in Ref. 2 (Ref. 3 presents essentially the same information as Ref. 2). This contention, however, does not find much support in facts, and, indeed, a close examination of Ref. 1 reveals very little resemblance to Ref. 2. Although both works are concerned with hybrid systems and consider stability analyses based on Liapunov's direct method, there the similarities end. Whereas the mathematical model of Ref. 1 is a solid body that is part rigid and part elastic, Ref. 2 considers rigid bodies with fluid-filled cavities. The real difference, however, lies not so much in the mathematical model, or the problem formulation, but in the method of approach to the stability problem. Indeed, Ref. 2 uses the standard Liapunov method to test the stability of a discrete system, whereas Ref. 1 develops a technique, based on the Liapunov direct method, to test the stability of a hybrid system. To be specific, Ref. 2 reduces the hybrid system to a discrete one by either considering cavities entirely filled with an ideal fluid and assuming that " . . . the motion of the fluid is completely defined by a finite number of variables" or by considering cavities partially or completely filled by an ideal or viscous fluid and assuming that " . . . in this case it is also possible to state the stability problem with respect to a finite number of variables by introducing certain quantities that integrally describe the motion of the fluid." To analyze such systems, Rumyantsev presents "Two theorems on stability with respect to a part of the variables, which can be regarded as modifications of the Lyapunov stability theorem." In conclusion, Rumyantsev interprets stability in a finite dimensional vector space consisting of the rigid body motion and a finite number of variables (depending on time but not on space) representing the fluid, thus avoiding many of the difficulties inherent in a stability analysis of truly hybrid systems.

The method of Ref. 1, by contrast, does not resort to any discretization scheme and interprets stability " . . . in a space S which can be regarded as the cartesian product of the finite dimensional vector space and the function space." The vector space is associated with the "rigid-body motion" and the function space with the motion of the elastic continuum. Since the system is hybrid, an expression which is both a function and l at the same time is considered for testing purposes; the expression is the system Hamiltonian. Difficulties caused by terms involving partial derivatives with respect to spatial variables in the Hamiltonian are circumvented by invoking certain properties of Rayleigh's quotient and devising a new testing function κ which is known to be smaller in value than the Hamiltonian. Moreover, defining a testing density function $\hat{\kappa}$ for every point of the elastic domain D_e, where $\hat{\kappa} = \kappa/D_e$, the sign properties of $\hat{\kappa}$ are checked at every point of D_e.

The author is confident that a more in-depth study will convince Rumyantsev that Ref. 1 does indeed contain many novel ideas not found anywhere else. The application of the techniques developed in Ref. 1 to test the stability of motion of rigid bodies with fluid-filled cavities is in the realm of possibility.

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Received April 21, 1971.

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Comment on "Spectroscopic Study of Ion-Neutral Coupling in Plasma Acceleration"

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MALLIARIS and Libby[1] have apparently overlooked an effect which may contribute appreciable errors to their measurements of axial velocity of neutrals in an MPD flow. Their velocity measuring technique cannot discriminate between particles which emanate from the thruster and identical particles which diffuse into the beam from the background. Both will be excited by collisional processes in the core of the beam and both will contribute light to the spectral line being observed. Their relative contributions will be in proportion to their relative densities. If the density of the background neutrals is not negligible compared to the density of the beam neutrals, a spectroscopic observation will yield a composite line.

The apparent doppler shift will give some sort of weighted average of the velocities of the two types of neutrals. Unless

Received March 1, 1971.

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they are efficiently entrained by the thruster exhaust the background neutrals, on the average, are stationary. The measurement will, therefore, underestimate the average velocity of the beam neutrals. Furthermore, at a given neutral flow rate, the density of the beam neutrals decreases with increasing axial velocity. The higher the true velocity, the more this measurement would underestimate it.

The magnitude of this effect is clearly appreciable in the experiments of Malliaris and Libby (configuration *a*). In the typical MPD conditions considered by the authors [after Eq. (14)], the beam density is taken as $2\,(10)^{14}$ cm^{-3} (ions plus neutrals). This corresponds to the lowest mass flow rate considered (10 mg/sec), so presumably the lowest tank pressure (10 μ) would be appropriate. The temperature of the background gas would be about the same as that of the tank wall or room temperature. Thus, for 10 $\mu \approx 10^{-5}$ atm, the background neutral density would be:

$$N \approx 10^{-5}N_o \approx 2.6(10)^{14}\text{cm}^{-3} \quad (1)$$

where N_o is the density at standard conditions. By the author's own arguments, the background gas would diffuse freely into the beam region. Since the estimated background density exceeds the estimated density of neutrals in the beam, a doppler shift measurement would significantly underestimate the velocity in this case.

The above effect may account for some of the qualitative difference between the results of Malliaris and Libby and those reported earlier by Sovie and Connolly.[2,3] Sovie and Connolly reported much higher neutral velocities than those observed by Malliaris and Libby. The Sovie and Connolly experiments, however, were performed at two orders of magnitude lower background pressure where the effect described above would be negligible.

References

[1] Malliaris, A. C. and Libby, D. R., "Spectroscopic Study of Ion-Neutral Coupling in Plasma Acceleration," *AIAA Journal*, Vol. 9, No. 1, Jan. 1971, pp. 160–167.

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Reply by Author to D. J. Connolly and R. J. Sovie

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CONNOLLY and Sovie[1] argue that the background particle density in our experiments[2] is comparable to the particle density in the beam. This might be so but, in an ammonia MPD flow, the beam fluid is much richer in atomic species (such as H and N), than the background fluid. The environmental background fluid (mainly NH_3, H_2 and N_2) diffusing into the beam, well downstream of the accelerator, has a much smaller chance of being dissociated because of the much milder conditions prevailing at these downstream stations. Thus, the presence of atomic species of background origin is fractionally negligible in the beam.

More important are the following experimental facts: in our experiments[2] we have used two configurations, a and b. The first is unfavorable to a strong ion-neutral coupling, while the opposite is true for the second. These configurations have been tried under experimental conditions which overlap over a certain range (see Table 1 of Ref. 2). In this range, whatever background effects are present in the expanded beam of configuration a should also be present in b. However, under otherwise identical conditions, the neutrals in a were found to be much slower than those in b. It follows that the background effect mentioned by Connolly and Sovie is not sufficiently strong to affect our measurements.

Received April 26, 1971.

References

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Comments on "Flow with $M_\infty = 1$ Past Thin Airfoils"

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IN a recent paper[1] a comparison of theoretical[2] and experimental[3] sonic pressure distributions is presented for airfoils having ordinates Z and chordwise location $(x/c)_{Z\,\max}$ of the maximum thickness given by

$$(Z/c) = A[(x/c) - (x/c)^n], \quad (x/c)_{Z\,\max} = (1/n)^{1/(n-1)} \ldots \quad (1)$$

and

$$(Z/c) = A[(1 - x/c) - (1 - x/c)^n], \quad (x/c)_{Z\,\max} = 1 - (1/n)^{1/(n-1)} \ldots \quad (2)$$

in which $A[\tau n^{n/(n-1)}]/[2(n-1)]$, c is the chord length and τ is the thickness–chord ratio. For maximum thickness locations rearward of $(x/c)_{Z\,\max} = 0.50$, the discrepancy between the theoretical and experimental distributions ceases to be small.

The validity of the experimental data quoted has been questioned by Thompson,[4] since they derive from wind-tunnel "bump" tests using a ventilated working section that may not have been adjusted precisely for interference free conditions. Also the chord length (3 cm) was relatively large in relation to the working section height (7 cm) and span (5 cm). These suspicions have been substantiated by measurements on symmetrical airfoils made in the 81 cm high, 53 cm span, Transonic Wind Tunnel at A.R.L., Melbourne.[5] The tunnel wall open area ratio and the airfoil chord-tunnel height ratio were varied in order to assess the extent of the interference. Airfoil chord length varied from 6.27 cm–20.32 cm, while thickness-chord ratio was held at 0.12 throughout. The maximum thickness location was $(x/c)_{Z\,\max} = 0.3$ (strictly 0.3011) corresponding to $n = 6$ in Eq. (2) above. Data for maximum thickness location $(x/c)_{Z\,\max} = 0.7$ corresponding to $n = 6$ in Eq. (1) above were obtained by reversing the airfoils.

Results estimated to be reasonably free of interference are presented in Fig. 1 as chordwise distributions of the transonic

Received May 14, 1971.

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